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Designation: AP

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Subject: Digital Signal Processing EC 601

DIGITAL FILTERS:

→ Discrete time LTI system.

→ Consist of Adders, Multipliers and shift Registers (delay elements)

TYPES OF DIGITAL FILTERS:

They are classified depending on number of sample points used to determine the unit sample response of LTI-DT system.

(a) Infinite Impulse Response filter (IIR)
(b) Finite Impulse Response filter (FIR)

→ If infinite number of sample points are used to determine unit sample response, then it is IIR filter. eg $h(n) = a^n u(n)$, $|a| > 1$

→ The IIR filters are of recursive type ie the present output sample depends on present input, past input samples and output samples.

$$y(n) = -\sum_{k=1}^{N-1} a_k y(n-k) + \sum_{k=0}^{M-1} b_k x(n-k)$$

a_k & b_k are coefficients of filter.

→ The impulse response of FIR filter is of finite duration eg $h(n) = \begin{cases} a^n u(n) & \text{for } n = 0 \text{ to } 5 \\ 0 & \text{otherwise} \end{cases}$

→ The FIR filters are of non recursive type ie the present output sample depends on present input

Unit	Topic	Page	Remark

sample and previous input samples.

$$y(n) = \sum_{k=0}^{M-1} b_k x(n-k)$$

Design of Digital IIR filters :

① Transform the given digital filter specifications to analog filter specifications.

② Design Analog filter H(s).

③ Transform the analog filter transfer function H(s) to a digital filter H(z).

The Transformation of an analog filter H(s) into a digital filter H(z) should posses the following properties:

① The $j\Omega$-axis of s plane should map onto the unit circle in the z-plane.

② The left half of s-plane maps on the inside of the unit circle in z-plane.

A stable analog filter will be transformed to a stable digital filter.

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Unit 5

IIR Filter designing using Impulse Invariant method :

We replace analog filter by digital filter. This is achieved if impulse response of digital filter resembles the sampled version of impulse response of analog filter.

$h_a(t)$ = Impulse response in time domain

$H_a(S)$ = Transfer fn of analog filter

$h_a(nT)$ = Sampled version of $h_a(t)$

$H(z)$ = z-transform of $h(nT)$.

Let the system transfer function of analog filter be $H_a(S)$

$$H_a(S) = \frac{A_1}{S-P_1} + \frac{A_2}{S-P_2} + \frac{A_3}{S-P_3} + \cdots$$

$$H_a(S) = \sum_{i=1}^{N} \frac{A_i}{S-P_i}$$ S – Laplace operator

Taking inverse Laplace transform

$$h_a(t) = \sum_{i=1}^{N} A_i e^{P_i t}$$

$h(n)$ can be obtained from $h_a(t)$ by replacing t by nT

$$h(n) = \sum_{i=1}^{N} A_i e^{P_i nT}$$ T → sampling time

$$H(z) = \sum_{n=0}^{\infty} h(n) z^{-n}$$

$$H(z) = \sum_{n=0}^{\infty} \left[\sum_{i=1}^{N} A_i e^{P_i nT} \right] \cdot z^{-n}$$

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$$H(z) = \sum_{i=1}^{N} A_i \sum_{n=0}^{\infty} e^{p_i n T} z^{-n}$$

$$= \sum_{i=1}^{N} A_i \sum_{n=0}^{\infty} \left(e^{p_i T} z^{-1}\right)^n$$

$$= \sum_{i=1}^{N} A_i \cdot \frac{1}{1 - e^{p_i T} z^{-1}}$$

this is the required transfer function of digital filter.

$$\frac{A_i}{s - p_i} \rightarrow \frac{A_i}{1 - e^{p_i T} z^{-1}}$$

The analog pole at $s = p_i$ is mapped into a digital pole at $z = e^{p_i T}$. Hence the analog poles and digital poles are related by relation

$$z = e^{sT} \rightarrow \text{laplace operator}$$

$$s = \sigma + j\Omega \rightarrow \text{analog freq. (rad/sec)}$$

σ → attenuation factor → (Neper freq in Nepers/sec)

$$z = r e^{j\omega} \rightarrow \text{digital freq.}$$

r → mag

$$z = e^{sT}$$

$$r e^{j\omega} = e^{(\sigma + j\Omega)T} = e^{\sigma T} \cdot e^{j\Omega T}$$

$$r = e^{\sigma T}, \quad \omega = \Omega T$$

∴ our analog pole is mapped to a place in the z-plane of magnitude $e^{\sigma T}$ and angle ΩT

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$$H(z) = \sum_{i=1}^{N} A_i \sum_{n=0}^{\infty} e^{p_i n T} z^{-n}$$

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$$z = e^{\underbrace{s}_{\text{laplace operator}}T}$$

$$s = \underbrace{\sigma}_{\text{attenuation factor} \rightarrow (\text{Neper freq in Nepers/sec})} + j \underbrace{\Omega}_{\text{analog freq. (rad/sec)}}$$

$$z = \underbrace{r}_{\text{mag}} e^{j\underbrace{\omega}_{\text{digital freq.}}}$$

$$z = e^{sT}$$

$$re^{j\omega} = e^{(\sigma + j\Omega)T} = e^{\sigma T} \cdot e^{j\Omega T}$$

$$r = e^{\sigma T}, \quad \omega = \Omega T.$$

$\therefore$ our analog pole is mapped to a place in the z-plane of magnitude $e^{\sigma T}$ and angle ΩT

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Consider the poles in left half of s-plane where $\sigma < 0$

$$r = e^{\sigma T}$$

$$\Rightarrow 0 < r < 1$$

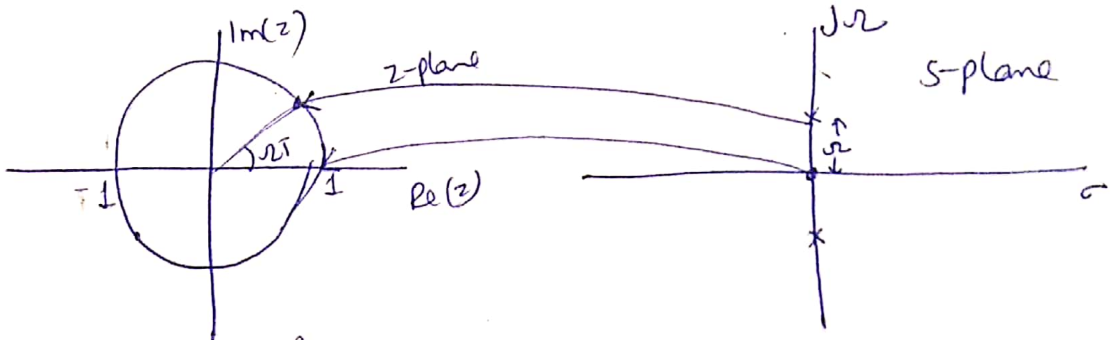
The poles map inside the unit circle

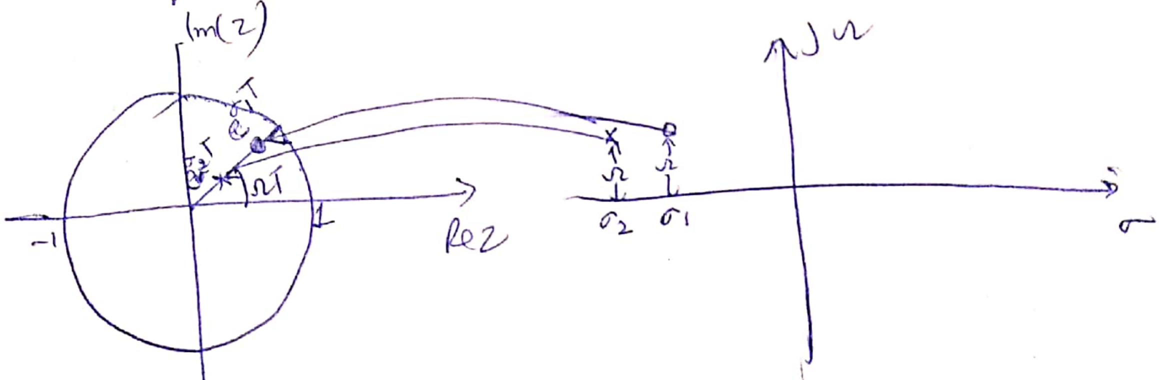
(b) If $\sigma > 0$, $r > 1$.

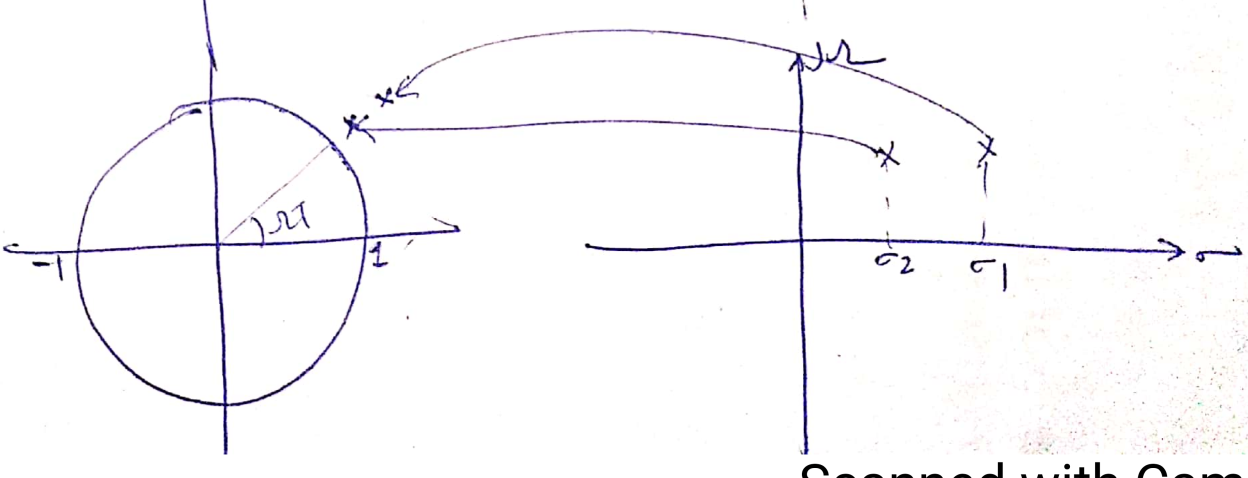
All poles in right half of s plane map to digital poles outside the unit circle

(c) If $\sigma = 0$ $\quad r = e^{0} = 1$

The $j\Omega$-axis is mapped into unit circle in z plane







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Impulse invariant transformation is not one to one mapping since many points in s-plane are mapped to a single point in z-plane, it is many to one mapping

$$s_1 = \sigma_1 + j\Omega$$

$$s_2 = \sigma_1 + j\left(\Omega + \frac{2\pi}{T}\right)$$

$$s_3 = \sigma_1 + j\left(\Omega + \frac{4\pi}{T}\right)$$

$$z_1 = e^{s_1 T} = e^{\sigma_1 T} \cdot e^{j\Omega T}$$

$$z_2 = e^{s_2 T} = e^{\sigma_1 T} \cdot e^{j(\Omega T + 2\pi)} \qquad (e^{j2\pi} = 1)$$

$$= e^{\sigma_1 T} \cdot e^{j\Omega T} = z_1$$

$$z_3 = e^{s_3 T} = e^{\sigma_1 T} \cdot e^{j(\Omega T + 4\pi)}$$

$$= e^{\sigma_1 T} \cdot e^{j\Omega T} = z_1$$

The relationship between analog & digital freq is $\boxed{\omega = \Omega T}$

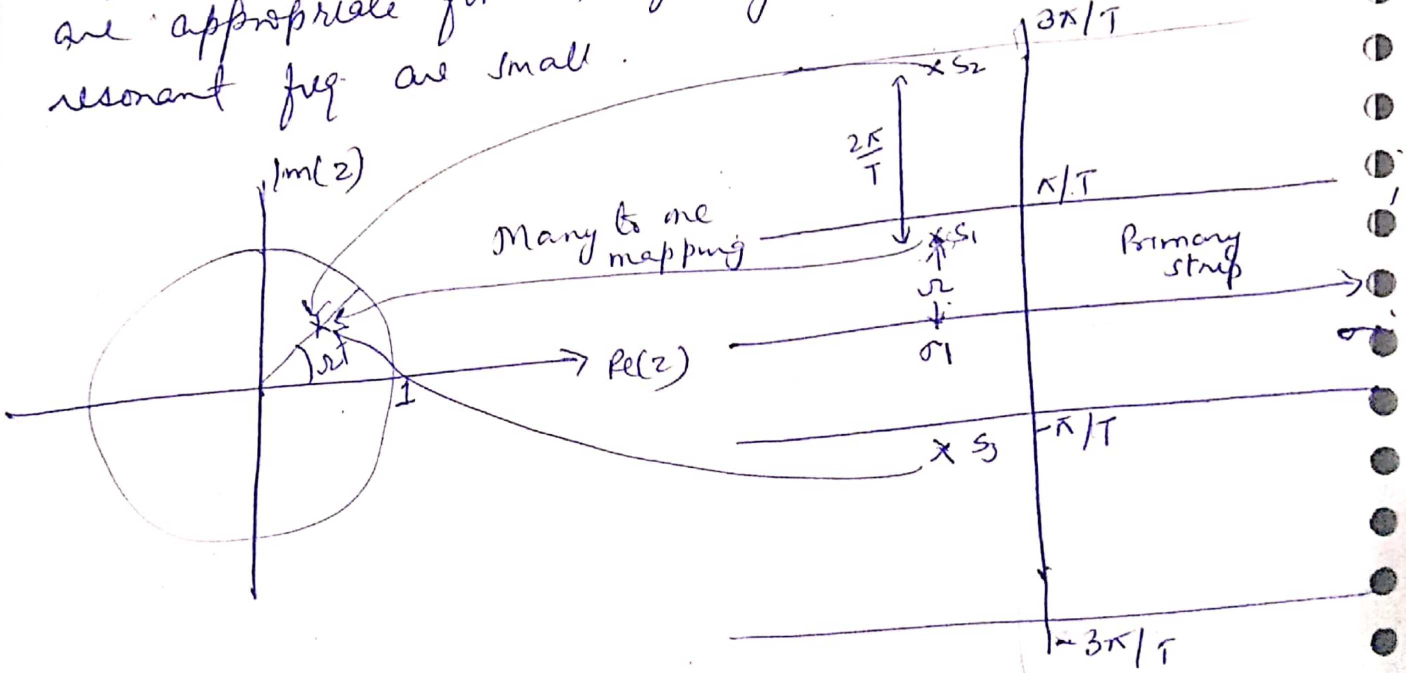
Consider a strip of width $\frac{2\pi}{T}$ across the s-plane. The strip of analog freq $\left[-\frac{\pi}{T} \le \Omega \le \frac{\pi}{T}\right]$ into $[-\pi \le \omega \le \pi]$.
(Ω on s-plane encircle the circle once)

Since $e^{j\Omega T} = e^{j\left(\Omega + \frac{2\pi}{T}\right)T}$, all freq $\Omega_0 \pm \frac{2\pi}{T}$ are mapped to same point in z-plane. As Ω increases from Ω_0 to $\Omega_0 + \frac{2\pi}{T}$, the freq ω increases from ω_0 to $\omega_0 + 2\pi$, and the segments of $j\Omega$ axis of length $\frac{2\pi}{T}$ thus map onto the unit circle over & over

A one to one mapping is possible only if ω lies in the principal range $-\pi \le \omega \le \pi$, corresponding to analog freq. range $-\frac{\pi}{T} \le \Omega \le \frac{\pi}{T}$.

The freq. interval $\frac{\pi}{T} \le \Omega \le \frac{3\pi}{T}$ also maps into the

-terval $-\pi \le \omega \le \pi$ and in general any freq interval $(2k-1)\frac{\pi}{T} \le \Omega \le (2k+1)\frac{\pi}{T}$ where k is integer will also map into interval $-\pi \le \omega \le \pi$ in z-plane.
Thus the mapping from analog. freq Ω to freq. variable ω in digital domain is many to one. This simple reflects the effects of aliasing. because of sampling of impulse response. The s plane poles having imaginary parts greater than π/T or less than $-\pi/T$ causes aliasing, when sampling analog signals.
$\therefore$ this method is unsuitable for implementing digital filter such as high pass filter. They are appropriate for design of LPF & BPF whose resonant freq are small.



9) For the analog transfer function

$$H(s) = \frac{2}{(s+1)(s+2)},$$ find $H(z)$ using impulse invariance method for sampling freq of 5 samples/sec.

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Soln – $H_a(s) = \dfrac{A}{s+1} + \dfrac{B}{s+2}$

$$A = (s+1) \cdot H_a(s)\Big|_{s=-1} = 1$$

$$B = -1$$

$$H_a(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

$$T = \frac{1}{f_s} = \frac{1}{5} = 0.2 \text{ sec}$$

$$\frac{1}{s-p_k} \rightarrow \frac{1}{1-e^{p_k T} z^{-1}}$$

$$\therefore H(z) = \frac{1}{1-e^{-1(0.2)} z^{-1}} - \frac{1}{1-e^{-2(0.2)} z^{-1}}$$

$$= \frac{1}{1-0.818 z^{-1}} - \frac{1}{1-0.67 z^{-1}}$$

$$H(z) = \frac{0.148 z}{z^2 - 1.48 z + 0.548}$$

Q Transform the analog filter Xfer function $H_a(s) = \dfrac{4s+7}{s^2+5s+4}$ using impulse into a digital filter $H(z)$ invariant method at $f_s = 2Hz$

Sol $H_a(s) = \dfrac{4s+7}{s^2+5s+4} = \dfrac{3}{s+4} + \dfrac{1}{s+1}$

$$T = \frac{1}{2} = 0.5 \text{ sec}$$

$$H(z) = \frac{3}{1-e^{-4T} z^{-1}} + \frac{1}{1-e^{-T} z^{-1}} = \frac{3}{1-e^{-2} z^{-1}} + \frac{1}{1-e^{-0.5} z^{-1}}$$

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$$H(z) = \frac{4 - 1.9549z^{-1}}{1 - 0.7419z^{-1} + 0.08217z^{-2}}$$

Q Convert the analog filter with system ~~transfer~~ function $H_a(s) = \frac{s+0.1}{(s+0.1)^2 + 9}$ into a digital IIR filter by means of impulse invariant method

$(s+0.1)^2 - (j3)^2$

$(s+0.1-j3)(s+0.1+j3)$

Sol

$P_1 = -0.1 + j3$

$P_2 = -0.1 - j3$

$$H_a(s) = \frac{s+0.1}{(s+0.1-j3)(s+0.1+j3)}$$

$$= \frac{A_1}{(s+0.1-j3)} + \frac{B}{(s+0.1+j3)}$$

$$A_1 = (s+0.1-j3) \cdot H_a(s)\Big|_{s=-0.1+j3}$$

$$= \frac{1}{2}$$

$$A_2 = (s+0.1+j3)\, H_a(s)\Big|_{s=-0.1-j3} = \frac{1}{2}$$

$$H_a(s) = \frac{\frac{1}{2}}{s+0.1-j3} + \frac{1/2}{s+0.1+j3}$$

$$H(z) = \frac{\frac{1}{2}}{1 - e^{-0.1T}e^{j3T}z^{-1}} + \frac{\frac{1}{2}}{1 - e^{-0.1T}e^{-j3T}z^{-1}}$$

$$= \frac{1}{2}\left(\frac{1 - e^{-0.1T}e^{-j3T}z^{-1} + 1 - e^{-0.1T}e^{j3T}z^{-1}}{(1-e^{-0.1T}e^{j3T}z^{-1})(1-e^{-0.1T}e^{-j3T}z^{-1})}\right)$$

$$= \frac{1}{2}\left(\frac{2 - 2e^{-0.1T}\cos 3T\, z^{-1}}{1 - 2e^{-0.1T}\cos 3T z^{-1} + e^{-0.2T}z^{-2}}\right)$$